Cambridge Assessment
Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION **D513/11**

PAPER 1 Morning

Practice paper Time: 75 minutes

Additional Materials: Answer sheet

6615122610

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.
There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 13 printed pages and 3 blank pages.

PV2

BLANK PAGE

1 It is given that the expansion of $(ax + b)^3$ is $8x^3 - px^2 + 18x - 3\sqrt{3}$, where a, b and p are real constants.

What is the value of p?

A $-12\sqrt{3}$

B $-6\sqrt{3}$

C $-4\sqrt{3}$

D $-\sqrt{3}$

E $\sqrt{3}$

F $4\sqrt{3}$

G $6\sqrt{3}$

H $12\sqrt{3}$

2 The expression $3x^3 + 13x^2 + 8x + a$, where a is a constant, has $(x + 2)$ as a factor.

Which one of the following is a complete factorisation of the expression?

A $(x + 2)(x - 1)(3x - 2)$

B $(x + 2)(x + 1)(3x - 2)$

C $(x + 2)(x + 1)(3x + 2)$

D $(x + 2)(x - 3)(3x + 2)$

E $(x + 2)(x + 3)(3x - 2)$

F $(x + 2)(x + 3)(3x + 2)$

[Turn over

3 A line is drawn normal to the curve $y = \frac{2}{x^2}$ at the point on the curve where $x = 1$.

This line cuts the x-axis at P and the y-axis at Q.

The length of PQ is

A $\frac{3\sqrt{5}}{2}$

B $\frac{3\sqrt{17}}{4}$

C $\frac{7\sqrt{17}}{4}$

D $\frac{35}{4}$

E $\frac{35\sqrt{5}}{2}$

F $\frac{3\sqrt{17}}{2}$

4 The sequence a_n is defined by the rule:

$$a_n = (-1)^n - (-1)^{n-1} + (-1)^{n+2} \text{ for } n \geq 1.$$

Find the value of

$$\sum_{n=1}^{39} a_n$$

A – 39

B – 3

C – 1

D 0

E 1

F 3

G 39

5 What is the total area enclosed between the curve $y = x^2 - 1$, the x-axis and the lines $x = -2$ and $x = 2$?

A $\frac{4}{3}$

B $\frac{8}{3}$

C 4

D $\frac{16}{3}$

E 12

F 16

6 P, Q, and R are each mixtures of red and white paint.

The percentage by volume of red paint in P is 30%.

The percentage by volume of red paint in Q is 20%.

The mixtures P, Q, and R are combined in the proportion 12 : 5 : 3 respectively.

If the resulting mixture contains 25% by volume of red paint, what percentage by volume of mixture R is red paint?

A 25%

B 23%

C $13\frac{1}{3}\%$

D $19\frac{1}{2}\%$

E $9\frac{3}{4}\%$

F It is impossible to achieve this result.

[Turn over

7 60% of a sports club's members are women and the remainder are men.

This sports club offers the opportunity to play tennis or cricket. Every member plays exactly one of the two sports.

$\frac{2}{5}$ of the male members of the club play cricket;

$\frac{2}{3}$ of the cricketing members of the club are women.

What is the probability that a member of the club, chosen at random, is a woman who plays tennis?

A $\frac{1}{5}$

B $\frac{7}{25}$

C $\frac{1}{3}$

D $\frac{11}{25}$

E $\frac{3}{5}$

8 Find the maximum angle x in the range $0° \leq x \leq 360°$ which satisfies the equation

$$\cos^2(2x) + \sqrt{3}\sin(2x) - \frac{7}{4} = 0$$

A 30°

B 60°

C 120°

D 150°

E 210°

F 240°

G 300°

H 330°

9 The line segment joining the points (3, 3) and (7, 5) is a diameter of a circle.

This circle is translated by 3 units in the negative x-direction, then reflected in the x-axis, and then enlarged by a scale factor of 4 about the centre of the resulting circle.

The equation of the final circle is

A $(x-2)^2+(y-4)^2=320$

B $(x-2)^2+(y+4)^2=320$

C $(x-2)^2+(y-4)^2=80$

D $(x-2)^2+(y+4)^2=80$

E $(x-2)^2+(y-4)^2=20$

F $(x-2)^2+(y+4)^2=20$

10 How many solutions does the equation $x\tan x=1$ have in the interval $-2\pi \leq x \leq 2\pi$?

A 0

B 1

C 2

D 3

E 4

F 5

G 6

[Turn over

11 The real roots of the equation $4^{2x} + 12 = 2^{2x+3}$ are p and q, where $p > q$.

The value of $p - q$ can be expressed as

A $\frac{3}{4}$

B 1

C 4

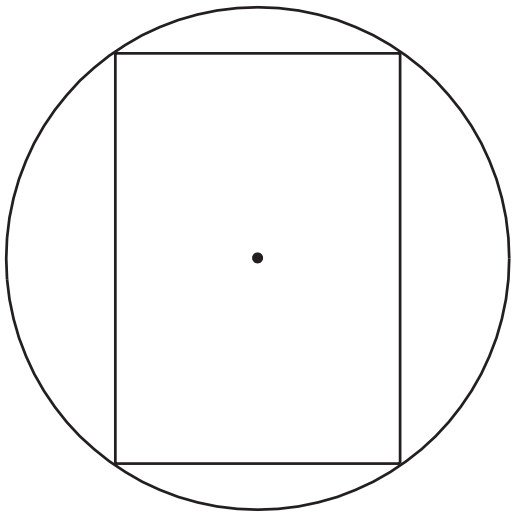
D $-\frac{1}{2} + \log_{10}\frac{3}{2}$

E $\frac{\log_{10} 3}{\log_{10} 4}$

F $\frac{\log_{10} 3}{\log_{10} 2}$

12 A right circular cylinder is contained within a sphere of radius 5 cm in such a way that the whole of the circumferences of both ends of the cylinder are in contact with the sphere.

The diagram shows a planar cross section through the centre of the sphere and cylinder.



[diagram not to scale]

Find, in cubic centimetres, the maximum possible volume of the cylinder.

A 250π

B 500π

C 1000π

D $\frac{250\sqrt{3}}{3}\pi$

E $\frac{500\sqrt{3}}{9}\pi$

F $\frac{1000\sqrt{3}}{9}\pi$

[Turn over

13 How many real roots does the equation $3x^5 - 10x^3 - 120x + 30 = 0$ have?

A 1

B 2

C 3

D 4

E 5

14 The terms of an infinite series S are formed by adding together the corresponding terms in two infinite geometric series, T and U.

The first term of T and the first term of U are each 4.

In order, the first three terms of the combined series S are 8, 3, and $\frac{5}{4}$

What is the sum to infinity of S?

A $\frac{32}{5}$

B $\frac{20}{3}$

C $\frac{64}{5}$

D $\frac{40}{3}$

E 16

F 32

15 The least possible value of the gradient of the curve $y = (2x + a)(x - 2a)^2$ at the point where $x = 1$, as a varies, is

A $-\frac{49}{4}$

B -8

C $-\frac{25}{4}$

D $\frac{7}{4}$

E $\frac{47}{16}$

16 Given the simultaneous equations

$$\log_{10} 2 + \log_{10}(y - 1) = 2 \log_{10} x$$

$$\log_{10}(y + 3 - 3x) = 0$$

the values of y are

A $\frac{5}{2} \pm \frac{3\sqrt{5}}{2}$

B $3 \pm \sqrt{3}$

C $7 \pm 3\sqrt{3}$

D $3, 9$

E $1, 13$

17 It is given that

$$y = (1 + 2\cos x)\cos 2x \qquad \text{for } 0 < x < \pi$$

The complete set of values of x for which y is negative is

A $0 < x < \frac{\pi}{4},\ \frac{2\pi}{3} < x < \frac{3\pi}{4}$

B $0 < x < \frac{\pi}{4},\ \frac{3\pi}{4} < x < \pi$

C $0 < x < \frac{2\pi}{3},\ \frac{3\pi}{4} < x < \pi$

D $\frac{\pi}{4} < x < \frac{2\pi}{3},\ \frac{3\pi}{4} < x < \pi$

E $\frac{\pi}{4} < x < \frac{2\pi}{3}$

F $\frac{\pi}{4} < x < \frac{3\pi}{4}$

18 The function $\frac{1-x}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$.

The complete set of values of x for which the function is decreasing is

A $x \leq -2,\ x > 0$

B $-2 \leq x < 0$

C $x \leq 1,\ x \neq 0$

D $x \geq 1$

E $-2 \leq x \leq 1,\ x \neq 0$

F $x \leq -2,\ x \geq 1$

19 The coefficient of x^3 in the expansion of $(1 + 2x + 3x^2)^6$ is equal to twice the coefficient of x^4 in the expansion of $(1 - ax^2)^5$.

Find all possible values of the constant a.

A $\pm 2\sqrt{2}$

B $\pm \sqrt{17}$

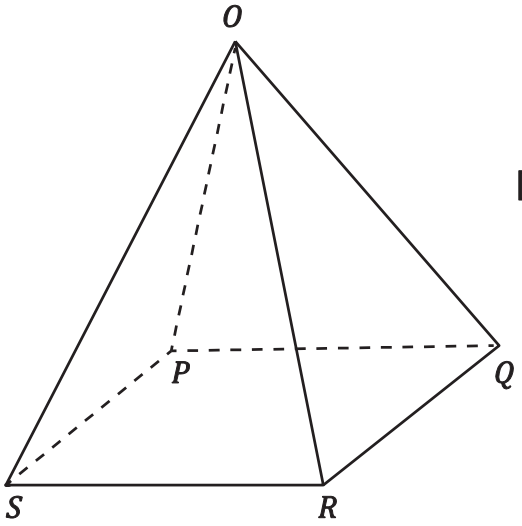
C $\pm \sqrt{34}$

D $\pm 2\sqrt{17}$

E There are no possible values of a.

[Turn over

20 The diagram shows a square-based pyramid with base $PQRS$ and vertex O. All the edges of the pyramid are of length 20 metres.



[diagram not to scale]

Find the shortest distance, in metres, along the outer surface of the pyramid from P to the midpoint of OR.

A $10\sqrt{5-2\sqrt{3}}$

B $10\sqrt{3}$

C $10\sqrt{5}$

D $10\sqrt{7}$

E $10\sqrt{5+2\sqrt{3}}$

END OF TEST

BLANK PAGE

BLANK PAGE

